Slow light in three-level cold atoms: a numerical analysis

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We investigate theoretically the slow group velocity of a pulse probe laser propagating through a cold sample and interacting with atoms in a three-level Λ configuration having losses towards external states. The EIT phenomenon produces very small group velocities for the probe pulse in presence of a strong coupling field even in presence of the population losses, as in an open three-level system. The group velocity and the transmission of the pulses are examined numerically as functions of several parameters, the adiabatic transfer, the loss rate, the modification of the atomic velocity produced within the cold sample. The conditions for a more efficient pulse transmission through the cold atomic sample are specified.

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I. INTRODUCTION

In a a three level Λ system, with a central excited level connected by electric dipole transitions to the two ground ones, electromagnetically-induced transparency (EIT) and reduction in group velocity are based on the low-frequency coherence created by the laser radiation [1,2]. One optical transition is driven by a strong laser, denoted as coupling laser, and the second transition is driven by a weak laser, the probe one. The probe laser absorption and dispersion are determined by the modifications in the populations and in the coherences produced by the coupling laser, and also by the low-frequency coherence between the ground states created by the simultaneous application of the coupling and probe lasers. For the three-level atomic Λ configuration, the absorption is decreased by the coherent trapping of the population in the ground state [3]. The probe light group velocity is greatly reduced by the very steep dispersion associated with the coherent population trapping narrow resonance. Remarkable results for the reduction of the group velocity in three-level systems were recently reported in cold samples [4–6] as well as in samples with thermal velocity distributions [7–9].

Often the real atomic transitions of quantum optics phenomena do not correspond to the simple theoretical models, and several multilevel atomic structures can be described as open systems, i.e., systems where population, whence coherent population trapping, is lost because of decay into sink levels not excited by the lasers. Even if the open features of a multilevel system may be eliminated through the application of a repumping radiation to deplete the sink state [10], they play an important role in most experiments. In this work we plan to investigate the propagation of slow light in an open atomic system for the specific conditions of the cold atom experiment by Lau et al [4]. We analyze a sample of cold sodium atoms laser excited in an open Λ scheme starting from two different hyperfine levels and with losses from the excited state towards another ground state. The experiment of [4] was performed with both a cold sodium atomic sample and a sodium condensate. However the atomic interactions are not included in our analysis [11].

The present work addresses several issues related to the slow light propagation. Our main aim is to determine the role of the open three-level system losses on the group velocity, probe pulse amplitude transmission and transmission bandwidth. In an open three-level system, any excited state occupation leads to a loss of the atomic population towards atomic states not excited by the laser light, whence to a modification of the slow light propagation. Our numerical simulations determine the laser parameters more appropriate to realize both efficient pulse slowing and transmission. We have specifically investigated if the STIRAP configuration [12,13] could be used to reduce the role of the excited state occupation in an open three-level system. In STIRAP the counterintuitive coupling/probe pulse sequence with detuned lasers produces a very efficient coherent-state atomic preparation from the initial ground state to the final ground state, and decreases the role of the excited state occupation with losses towards external levels. We have verified that, even if the counterintuitive pulse sequence enhances the probe pulse transmission, the EIT conditions for slow light do not correspond precisely to those for the STIRAP process. We investigate also the influence of the adiabatic transfer conditions [14] on the pulse transmission: a larger probe pulse amplitude transmission is obtained for a larger temporal width of the probe pulse, because that width modifies the adiabaticity condition.

The transmission of a slow probe pulse depends on the decay rate of the coherence between the lower levels of the Λ system. An efficient pulse propagation, *i.e.* a small absorption coefficient, is obtained by reducing the coherence decay rate. In a sample of room temperature atoms, collisions and transit time are the main contributions to that decay rate. We examine the influence of a coher-

ence relaxation rate on the pulse transmission and on the velocity reduction. In a cold atom sample, the previous contributions to the coherence decay rate are greatly decreased. However in a cold sample with long interaction times the kinetic energy motion of the atoms represents another important contribution to the coherence decay rate. The forces by the laser on the atoms modify the atomic momentum and whence the parameters of the laser atom interaction. We investigate the pulse propagation taking into account the atomic momentum modifications occurring in the laser interaction. The importance of light forces on atoms in the case of slow light propagation has been recently investigated [15] for the configuration where a slowly propagating laser pulse produces a force acting on a second atomic species contained within the propagation medium. In the present work the light forces act on the same atoms producing the slow light process. Thus we investigate the limitations imposed on the slow light propagation by the light forces acting on the momentum of the atoms producing the light slowing.

The present analysis makes use of the time/space dependent numerical solution the Optical Bloch Equations (OBE) of an open three-level system. Section II introduces the three-level system and the coupling/probe laser propagation. Section III presents the numerical results for the propagation of a laser pulse through a cold sample of open three-level atoms in the presence of a coupling laser. Section IV investigates the role of the atomic momentum. A conclusion completes our work in Section V.

II. THREE-LEVEL SYSTEM AND LASER PROPAGATION

A. Atoms

We consider a cold sample of open three-level atoms interacting with two laser fields in the Λ configuration. The interaction scheme is shown in Fig. 1. The ground (or metastable) states $|\mathrm{c}\rangle$ and $|\mathrm{p}\rangle$, with energies respectively E_{c} and E_{p}, are excited to a common state $|\mathrm{e}\rangle$, with energy E_{e}, by two laser fields of frequencies ω_{c}, ω_{p}, electric field amplitudes $\mathcal{E}_{\mathrm{c}}$, $\mathcal{E}_{\mathrm{p}}$, and wavevectors $k_\alpha = \omega_\alpha/c$, with $(\alpha = \mathrm{c},\mathrm{p})$, c being light speed in vacuum. As standard in laser without inversion and similar processes [1,2], the laser fields are indicated as coupling (c) and probe (p) lasers. The laser detunings are denoted by $\delta_\alpha = \omega_\alpha - \omega_{\mathrm{e}\alpha}$ and the Raman detuning from the two-photon resonance is $\delta_R = \delta_{\mathrm{c}} - \delta_{\mathrm{p}}$. The Rabi frequencies are given by

$$\Omega_\alpha = \frac{\mathcal{D}_{\mathrm{e}\alpha}\mathcal{E}_\alpha}{\hbar}, \tag{1}$$

with $\mathcal{D}_{\mathrm{e}\alpha}$ the atomic dipole moment, supposed to be real for the sake of simplicity. The evolution of the atomic density matrix ρ is described by the OBE for an open system [16]. We assume the excited state relaxation, denoted by A, due to spontaneous emission, as for a dilute atomic sample. In the open system the A rate is composed by the rates Γ_{c} and Γ_{p} for the decays into the ground states, and by the decay Γ_{out} into a sink state $|out\rangle$, not excited by the lasers

$$A = \Gamma_{\mathrm{c}} + \Gamma_{\mathrm{p}} + \Gamma_{\mathrm{out}}. \tag{2}$$

Thus the total decay rate of the excited state population, denoted by Γ_{e}, is equal to A, and the decay rate of the optical coherences, ρ_{ep} and ρ_{ec}, is equal to $A/2$. For a given open system the application of a repumping laser, as performed in [10], eliminates the decay channel out of the system. Thus we may compare the slow light propagation in an open system characterized by the above relaxation rates, to the propagation in the closed system produced by the repumping laser. That closed system is characterized by the following population decay for the excited state:

$$\Gamma_{\mathrm{e}}^{'} = \Gamma_{\mathrm{c}} + \Gamma_{\mathrm{p}}, \tag{3}$$

while in the presence of repumping the decay rate of the optical coherences remains $A/2$. In our analysis we will introduce also a decoherence rate γ_{cp} for the ground state coherence ρ_{cp}.

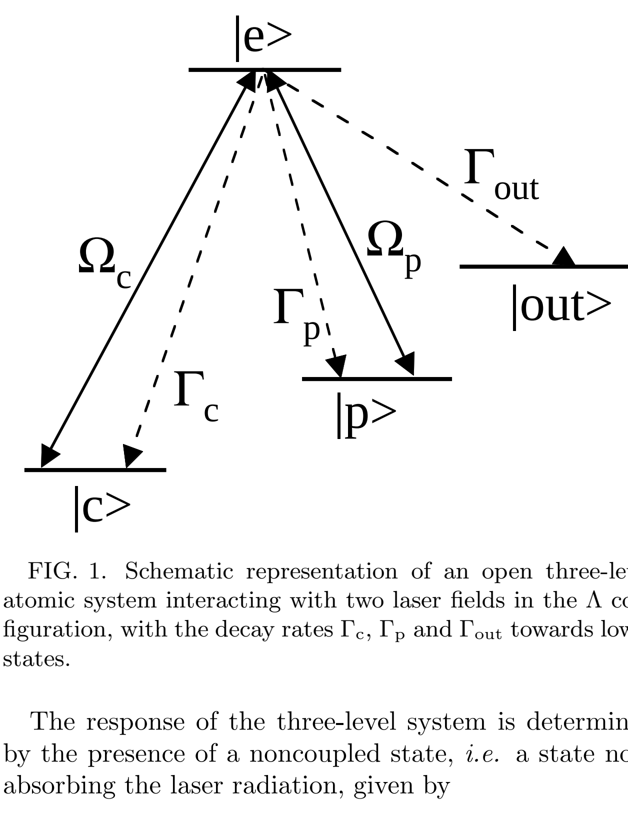

FIG. 1. Schematic representation of an open three-level atomic system interacting with two laser fields in the Λ configuration, with the decay rates Γ_{c}, Γ_{p} and Γ_{out} towards lower states.

The response of the three-level system is determined by the presence of a noncoupled state, *i.e.* a state nonabsorbing the laser radiation, given by

$$|\mathrm{NC}(t)\rangle = \frac{1}{\Omega}\left(\Omega_{\mathrm{p}}|c\rangle - \Omega_{\mathrm{c}} e^{i(\omega_{\mathrm{p}}-\omega_{\mathrm{c}})t}|p\rangle\right) e^{-i\frac{E_c t}{\hbar}}. \tag{4}$$

where Ω is defined by

$$\Omega = \sqrt{|\Omega_{\rm p}|^2 + |\Omega_{\rm c}|^2}. \tag{5}$$

The decoherence rate $\gamma_{\rm cp}$ couples $|{\rm NC}(t)\rangle$ to its orthogonal state $|{\rm C}(t)\rangle$.

In both closed and open three-level systems an optical pumping preparation of the noncoupled state, taking place within few excited state lifetimes, requires a significant occupation of excited state population [3]. While in a closed system all the atomic population falls into the noncoupled state without loss of atomic population, for an open system a decay out of the three-level system takes place. For an open system, the occupation of the ground states $|c\rangle$ and $|p\rangle$ at long interaction times is different from zero only at small Raman detunings, where coherent population trapping is efficient. The analysis of Ref. [16] indicates that at long interaction times the slope of ${\rm Re}(\tilde{\rho}_{\rm ep})$ at $\delta_{\rm R} \neq 0$ reaches a constant nonzero value. In effect around $\delta_{\rm R} = 0$ the population is trapped in the nonabsorbing state up to an interaction time Θ dependent on $\delta_{\rm R}$ [16,17]. A constant, and slow, group velocity is realized because the slope of the dispersion around $\delta_{\rm R} = 0$ does not change significantly for an increasing interaction time Θ, even if the interval of sharp variation of the dispersion decreases with Θ. However, the loss rate out of the three-level system increases with the Raman detuning, and the transparency window narrows for increasing interaction time.

B. Lasers

To describe the propagation of laser pulses through a cold sample of open three-level atoms, the electric field amplitudes, whence the Rabi frequencies, are assumed dependent on the time t and the coordinate z along the propagation direction. For slowly varying electric field amplitudes, the Maxwell equations for the coupling/probe envelope Rabi frequencies expressed in terms of the pulse-localized coordinates z and $\tau = t - z/c$, $\Omega_{\rm p}(z,\tau)$ and $\Omega_{\rm c}(z,\tau)$ reduce to [18]

$$\frac{\partial}{\partial z}\Omega_\alpha(z,\tau) = i\kappa_\alpha \tilde{\rho}_{{\rm e}\alpha}(z,\tau) \tag{6}$$

with $\alpha = (c,p)$ and where the parameter κ_α is given by

$$\kappa_\alpha = \frac{\omega_\alpha N \mathcal{D}^2_{{\rm e}\alpha}}{c\epsilon_0\hbar} \tag{7}$$

N being the atomic density and ϵ_0 the vacuum susceptibility. The Beer's absorption length for the probe field $z_{\rm p}$ in the absence of coupling laser is

$$\zeta_{\rm p} = \frac{A}{\kappa_{\rm p}}. \tag{8}$$

In the ideal case, the solution of OBE and of Eq. (6) leads to a shape-invariant propagation of the probe pulse described through the dependence $\Omega_{\rm p}(t - \frac{z}{v_{\rm g}})$, $v_{\rm g}$ being the probe pulse group velocity.

OBE for the density matrix and Eqs. (6) for the laser propagation were solved numerically for an initial Gaussian pulse on the probe transition

$$\Omega_{\rm p}(z=0,t) = \Omega_{\rm op}\exp\left(\frac{-t^2}{2T^2}\right). \tag{9}$$

In the simulation the coupling laser, with constant Rabi frequency $\Omega_{\rm c}$, was assumed switched on at earlier times $t \leq 0$, in order to realize a counterintuitive pulse sequence. The atoms were initially prepared in the ground $|{\rm p}\rangle$ state. Owing to the initial condition and the counterintuitive pulse sequence, the atoms were initially prepared in the noncoupled state.

Because of the time dependence of the Rabi frequencies in the noncoupled state of Eq. (4), the adiabatic coherent preparation requires a proper choice of the timescales [12–14]. In STIRAP the conditions for the atoms in the initial $|p\rangle$ state to remain in the noncoupled state following the adiabatic evolution are [12]

$$\Omega_\alpha \geq A; T \geq 1/\Omega_\alpha, \tag{10}$$

with $\alpha = (c,p)$. The first inequality, valid for resonant lasers, imposes a minimum Rabi frequency. Larger Rabi frequencies are required for detunings $\delta_{\rm c}$ and $\delta_{\rm p}$ different from zero [12]. The second inequality imposes a pulse width long compared to the inverse of the Rabi frequencies. We have verified numerically the role of both conditions on the slow light propagation. However it should be noted that the STIRAP conditions are required for an adiabatic transfer of the atomic population, while for slow light the atomic response is not important, and the only request is on the probe pulse propagation. For instance in the limit of a very small $\Omega_{\rm p}$ Rabi frequency, a negligible STIRAP transfer is realized, still a slow light propagation may be produced. In Ref. [14] where the formstable pulse propagation was linked to the preservation of population in the noncoupled state, the condition for a small nonadiabatic coupling between noncoupled $|{\rm NC}(t)\rangle$ and coupled $|{\rm C}(t)\rangle$ states was expressed through the following Rabi frequency coupling Ω_-:

$$\Omega_- = \frac{\dot{\Omega}_{\rm c}\Omega_{\rm p} - \dot{\Omega}_{\rm p}\Omega_{\rm c}}{\Omega^2}, \tag{11}$$

where the dot denotes the time derivative. On the basis of this Rabi coupling, the adiabaticity condition required for a formstable probe pulse propagation was written [14]

$$\Omega_- \ll \Omega \tag{12}$$

This relation, and the second one of Eq. (10), should be satisfied for the slow light propagation.

The probe Rabi frequency transmitted after the propagation through the cold gas medium was determined

numerically, with the pulse delay $\tau_{\rm d}(z)$ at the given spatial position z defined by

$$\tau_{\rm d}(z)=\frac{\int_{-\infty}^{+\infty}\tau|\Omega_{\rm p}(z,\tau)|^2d\tau}{\int_{-\infty}^{+\infty}|\Omega_{\rm p}(z,\tau)|^2d\tau}. \tag{13}$$

Finally the group velocity was calculated [19]

$$v_{\rm g}=\frac{c}{1+\frac{c\tau_{\rm d}(z)}{z}}. \tag{14}$$

The bandwidth $\Delta\omega$ of the EIT transparency window determines the spectral components of the probe pulse propagating deep into the atomic medium without shape distortion. The EIT window was explicitly investigated in the applications of slow light to quantum entanglement and stopping of light [20]. The transparency window for both closed and open systems depends on the decay of the noncoupled state. For a closed system with $\gamma_{\rm cp}=0$ at $\delta_{\rm R}=0$ the transparency window is given by

$$\Delta\omega=\frac{\Omega^2}{A}+\frac{1}{\Theta}, \tag{15}$$

whence the window decreases towards a limiting value at large interaction times Θ. For an open system that window is [16]

$$\Delta\omega=\frac{\Omega^2}{2\sqrt{A\Theta}}\sqrt{\left(1+\frac{\Gamma_{\rm p}}{\Gamma_{\rm out}}\right)\frac{1}{\Omega_{\rm p}^2}+\left(1+\frac{\Gamma_{\rm c}}{\Gamma_{\rm out}}\right)\frac{1}{\Omega_{\rm c}^2}}. \tag{16}$$

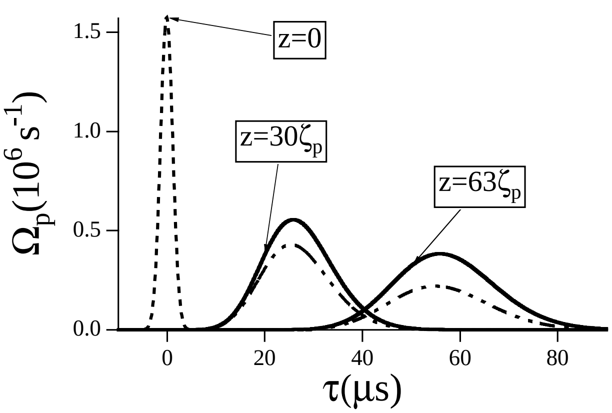

FIG. 2. Probe Rabi frequency $\Omega_{\rm p}(z,\tau)$ as a function of the pulse-localized time τ at different penetration distances in the conditions of ref. [4], with $\Gamma_{\rm c}=A/3$, $\Gamma_{\rm p}=A/2$, and $\Gamma_{\rm out}=A/6$. The dashed line represents a reference probe pulse with no atoms in the propagation medium, scaled down by a factor two. The continuous and dashed-dotted lines are pulses for propagation through the medium at $z=30\zeta_{\rm p}$ and at $z=63\zeta_{\rm p}$. Continuous lines for $\gamma_{cp}=0$ and dashed-dotted lines for $\gamma_{cp}=10^4$ s^{-1}. Initial Gaussian probe pulse width $T=80/A$, and initial coupling laser Rabi frequency $\Omega_{\rm c}=0.18A$.

For the probe pulse propagation, where the interaction time Θ can be approximated by the pulse duration T, in order to have a formstable pulse the transmission window should be larger than the Fourier frequency distribution of the pulse

$$\Delta\omega\gg 1/T. \tag{17}$$

For an open system with bandwidth given by Eq. (16), Eq. (17) imposes limitations on the Rabi frequencies. That condition is satisfied by the parameters of typical slow light pulses [4–9]. Only for very short pulses, or very weak coupling lasers, the EIT bandwidth could produce a large distortion of the pulse shape propagating through a three-level open system. We will derive in the next Section that, owing to Eqs. (10) and (12), a better transmission of the slow light pulses is obtained by increasing the pulse duration T.

III. PROBE TRANSMISSION

Numerical results for the propagation of the probe pulse within a sample of open three level atoms are shown in Fig. 2. We used the parameters of the experiment in Ref. [4], *i.e.* Na D_2 line parameters ($A=2\pi\cdot 5.9$ MHz, $\lambda=589.0$ nm), cold atom density of $3.3\cdot10^{12}$ atoms/cm^3, propagation length through the medium up to a distance z equal to 63 times the Beer's length $\zeta_{\rm p}$, initial Rabi frequencies $\Omega_{\rm op}$ and $\Omega_{\rm c}$ around a tenth of A. The numerical results show a pulse peak height decreasing with the penetration distance, and a probe pulse propagating with a slow velocity. We calculated from the data of Fig. 2 the slow velocities shown in Fig. 3, i.e. in the range of the values measured in ref. [4]. In order to compare these results *quantitatively* with the response of a closed system, the atomic parameters for the closed system should be chosen carefully. We may introduce a correspondence between closed and open systems by means of a repumping laser. As discussed previously, we compared the slow light propagation in an open system characterized by the $\rho_{\rm ee}$ decay rate A of Eq. (2), to that of a closed system characterized by the decay rate $\Gamma_{\rm e}'$ of Eq. (3), the same optical coherence decay rate $A/2$ applying to both systems. Numerical analyses for a closed system corresponding to the open system of Fig. 2 have shown that the same pulse transmission and same slow light velocity are obtained for the two systems. In fact the pulse propagation of Fig. 2 was examined for a counterintuitive coupling/probe pulse sequence and with laser parameters producing a very small excited state occupation, whence a small role of the $\Gamma_{\rm out}$ loss process.

We have examined the probe transmission modification produced by the decoherence rate $\gamma_{\rm cp}$. The decoherence process decreases the $|{\rm N}C(t)\rangle$ occupation, whence decreases the probe transmission. We have verified that

for $\gamma_{\rm cp}$ larger than $0.001A$, the probe transmission is well described through the EIT absorption length $\zeta_{\rm p}^{\rm EIT}$ given by the following formula [21]:

$$\zeta_{\rm p}^{\rm EIT} = \frac{A}{\kappa_{\rm p}} \left[\frac{\Omega_{\rm c}^2}{2\gamma_{\rm cp} A} + 1 \right] \tag{18}$$

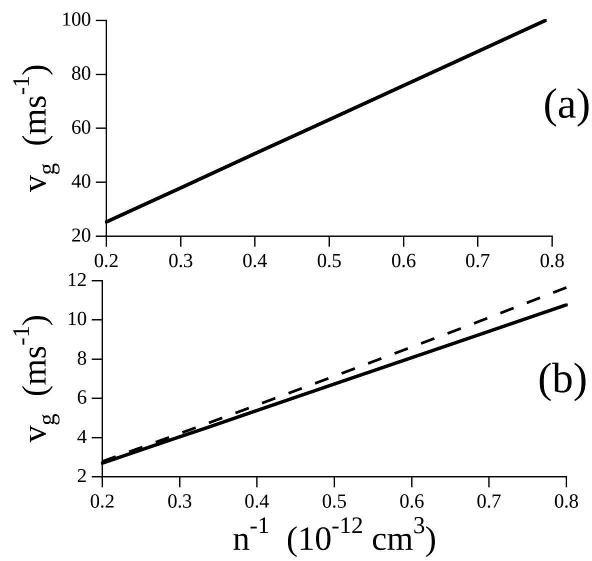

FIG. 3. Probe-pulse velocity versus the inverse of the atomic density. The data are in (a) for coupling intensity of 12 mW/cm^2 corresponding to coupling Rabi frequency $\Omega_{\rm c}$ =0.56 A, and in (b) for a coupling intensity of 3 mW/cm^2 corresponding to a coupling Rabi frequency $\Omega_{\rm c}$ =0.18 A. The continuous line represents results for a simulation not including the atomic momentum, while results of the dashed line include in the simulation the modifications of atomic momentum produced by the light forces. In (a) continuous and dashed lines cannot be distinguished on the scale of the plot.

Note that Eq. (18) predicts no attenuation for the case of $\gamma_{\rm cp} = 0$. On the contrary the numerical results of Fig. 2 evidence an attenuation of the probe pulse even in the ideal case of zero ground state decoherence. In effect for the parameters of Fig. 2 the adiabaticity condition of Eq. (12) required for the complete validity of the EIT description is not fully satisfied. The adiabaticity condition was calculated for the laser pulses of Fig. 2, as presented in Fig. 4(a). Ω_- strongly depends on the value of T, the probe pulse duration. Better adiabatic conditions, whence a larger probe pulse transmission, are realized at larger T values, with longer duration pulses, as shown in Fig. 4(b). We verified that the probe-light slow velocity has a weak dependence on the pulse length T, as in [16]. That weak dependence confirms that the dispersion slope of $Re(\rho_{\rm ep})$ versus the laser detuning reaches a constant value at large interaction time.

We have examined the dependence of the slow light velocity on the atomic density n, in order to simulate the dependence on the cold atom temperature investigated in [4]. In the experimental investigation the atomic density n was varied with the sample temperature. Our results for the slow light velocity are shown in Fig. 3 for two different values of the coupling laser parameters $\Omega_{\rm c}$. A linear dependence of the velocity on the inverse of the atomic density is derived from the data, with values between few m/s and 100 m/s depending on the atomic density and the coupling laser Rabi frequency. The lowest velocities are reached by decreasing the coupling laser Rabi frequency $\Omega_{\rm c}$. For the dense media required for cold atom slow light propagation, the reabsorption of the spontaneously emitted photons, neglected in our theoretical analysis, may play an important role. That reabsorption is greatly reduced in the cigar shape geometries of cold atomic samples [22].

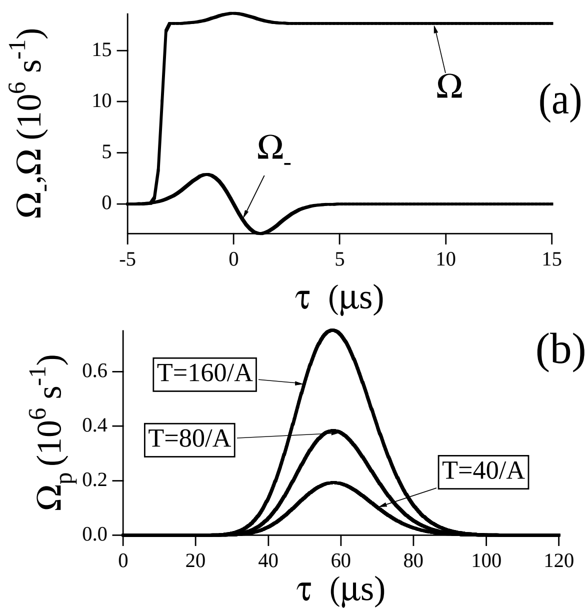

FIG. 4. In (a) non-adiabaticity test comparison between Ω_- and Ω for $T = 80/A$ at $z = 0$. In (b) probe Rabi frequency $\Omega_{\rm p}$ as a function of the pulse-retardation time t at $z = 63\zeta_{\rm p}$ in the open system of Fig. 2 with different temporal widths of the probe pulse. Higher transmission pulses are obtained at $T = 160/A$ decreasing transmission at $T = 80/A$ and $T = 40/A$.

The numerical simulation allowed us to verify the presence of adiabatons on the coupling laser, *i.e.* modifications in the coupling laser intensity propagating through the atomic sample synchronously with the probe laser [23]. These modifications are small, less than a ten percent of the coupling laser intensity. Their observation in experiments as those of Refs. [4–9] would represent a confirmation of the coupling/probe matched pulse propagation.

The probe pulse propagation was examined also for the case of laser detunings $\delta_{\rm c}$, $\delta_{\rm p}$, different from zero, satisfying the Raman resonance condition $\delta_{\rm R} = 0$. The results

of such simulation are shown in Fig. 5. For laser detunings different from zero, the probe field Rabi frequency acquires components in phase and out of phase with the initial probe laser field, and both components, $Re(\Omega_{\rm p})$ and $Im(\Omega_{\rm p})$ contribute to the pulse propagation. The absolute value $|\Omega_{\rm p}|$ is plotted in Fig. 5(a) while the separate in-phase and out-phase components are shown in Fig. 5(b). The individual in-phase and out-phase components acquire complicate pulse shapes, even if the absolute Rabi frequency propagates preserving the initial shape, apart from an attenuation and a broadening. The presence of a laser detuning imposes stronger adiabaticity conditions, not well satisfied by the weak Rabi coupling frequencies required for slow light propagation. In fact, comparing the results of Fig. 5(a) with those of Fig. 2, it appears that even for a small detuning the pulse shape of a slow pulse is less preserved. Increasing the laser detuning, the adiabaticity condition can be satisfied increasing the coupling laser Rabi frequency, at expenses of an increase of the slow velocity associated with the probe pulse.

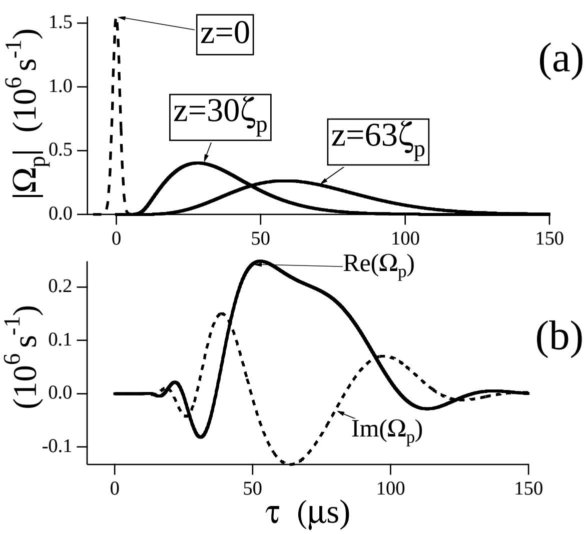

FIG. 5. Propagation of a probe pulse slow light for coupling/probe lasers detuned from the upper $|e\rangle$ level, maintaining the Raman resonance condition $\delta_{\rm R} = 0$, i.e. $\delta_{\rm c} = \delta_{\rm p} = -A$, and other parameters as in Fig. 2. In (a) absolute value of the probe Rabi frequency, $|\Omega_{\rm p}|$, versus the pulse-localized time τ. As in Fig. 2 the dashed line indicates a reference pulse. Continuous lines for different propagation distances through the medium. In (b) in-phase and out-phase components of the probe Rabi frequency, $Re(\Omega_{\rm p})$ and $Im(\Omega_{\rm p})$ respectively, at propagation distance $z = 63\zeta_{\rm p}$.

IV. ATOMIC MOMENTUM

While the previous analysis neglected the atomic motion, in a sample of cold atoms the modifications of the atomic momentum produced by the laser interactions should be taken into account. The atomic states will be classified by the internal variable $|\alpha\rangle$ with $(\alpha = c, p, e)$ and by the external continuous variable $\vec{p}$ of the atomic momentum. For an atom initially in the $|p\rangle$ state with momentum $\vec{p} = 0$, the Raman process of absorption and stimulated emission transfers the atoms from state $|p, \vec{p} = 0\rangle$ into the state $|c, \vec{k}_{\rm p} - \vec{k}_{\rm c}\rangle$. Including the atomic momentum the noncoupled state at $t = 0$ becomes

$$|{\rm NC_m}(t=0)\rangle\rangle = \frac{1}{\Omega}\left(\Omega_{\rm c}|p,0\rangle - \Omega_{\rm p}|c, \vec{k}_{\rm p} - \vec{k}_{\rm c}\rangle\right). \quad (19)$$

The kinetic energy Hamiltonian operator $H_{\rm k} = p^2/2M$, M being the atomic mass, couples the $|{\rm NC_m}\rangle$ state to its orthogonal one $|{\rm C_m}\rangle$ with a rate $\gamma_{\rm k}$ given by

$$\gamma_{\rm k} = \frac{\langle {\rm C_m}|H_{\rm k}|{\rm NC_m}\rangle}{\hbar} = \frac{\hbar\left(\vec{k}_{\rm c} - \vec{k}_{\rm p}\right)^2}{2M}\frac{\Omega_{\rm p}^2}{\Omega^2} \quad (20)$$

This rate $\gamma_{\rm k}$ decreases the ground state atomic coherence, and whence modifies the slowing down process of the light pulse. For $\vec{k}_{\rm c} \sim \vec{k}_{\rm p}$, a rate $\gamma_{\rm k}$ equal zero is realized in the coupling/probe copropagating laser configuration, and this result applies also for an initial atomic momentum $\vec{p}$ different from zero, as in the experiments of Refs. [7,8]. Instead the $\vec{k}_{\rm c} \sim -\vec{k}_{\rm p}$ counterpropagating configuration was used in the laser cooling based on velocity-selective coherent population trapping [24]. For the experiment of [4] with orthogonal propagation directions for coupling and probe lasers, and the parameters of Fig. 2, the decoherence rate $\gamma_{\rm k}$ is equal to 10^4 s^{-1}, close to the value used in the numerical analyses of Fig. 2 with $\gamma_{\rm c} \neq 0$. It should be noted that the recoil frequency for sodium is $\omega_{\rm R} = \hbar k^2/2M = 1.7\times10^5$ s^{-1}, but in Eq. (20) the $(\Omega_{\rm p}/\Omega)^2$ factor produces a smaller $\gamma_{\rm k}$.

In order to test the influence of the atomic momentum decoherence on the slow light production, we solved numerically the generalized OBE, *i.e.* the density matrix equations including the atomic momentum [24], for the parameters of the experiment in [4], i.e. with orthogonal $\vec{k}_{\rm c}$ and $\vec{k}_{\rm p}$. The numerical results for the slow light velocity including the atomic momentum in the propagation are shown in Fig. 3. The linear dependence of $v_{\rm g}$ on $1/n$ remains also when the atomic momentum is included in the analysis. The modification of the slow light velocity produced by the atomic momentum is around ten percent at the smaller coupling laser Rabi frequency.

The modifications on the atomic momentum produced by the laser pulse interaction originate from the forces acting on the atoms. The longitudinal force, along the laser propagation direction, on the atom of the slow light medium contains both a radiation-pressure dissipative term and a dipole gradient reactive term [25]. The dissipative radiation-pressure force $F_{\rm rp}$ acts on the atoms for both resonant and non resonant probe lasers, while the reactive dipole force $F_{\rm dip}$ is present only for $\delta_{\rm p} \neq 0$.

These forces are determined by the optical coherence of the atomic dipole matrix [26]:

$$F_{\mathrm{rp}}(z,t) = -i\frac{\hbar}{2}k_{\mathrm{p}}\Omega_{\mathrm{p}}(z,t)\rho^*_{\mathrm{pe}}(z,t) + c.c.$$
$$F_{\mathrm{dip}}(z,t) = \frac{\hbar}{2}\frac{\delta\Omega_{\mathrm{p}}}{\delta z}\rho^*_{\mathrm{pe}} + c.c \tag{21a}$$

where, owing to the counterintuitive pulse sequence, and the orthogonal $\vec{k}_{\mathrm{c}}$, $\vec{k}_{\mathrm{p}}$ geometry of Ref. [4], only the probe laser contributes to the force along the z axis. These forces, calculated for the propagating pulses of the previous figures, are plotted versus time at $z = 0$ in Fig. 6 for the case of $\delta_{\mathrm{c}} = \delta_{\mathrm{p}} = -A$. From the figure it appears that for the chosen laser detuning the radiation pressure force is larger than the dipole force by few orders of magnitude. Similar values were obtained for the radiation pressure force for a probe laser in resonance, when the dipole force is equal zero. From the point of view of the atomic response, the total modification of the atomic momentum depends on the integral of the force over the interaction time, i.e. the pulse duration time. Because the radiation pressure force has an antisymmetric dependence on the time, its integral, whence the modification of the atomic momentum, is small. We have verified that for same laser parameters, detunings and Rabi frequencies, for instance those of Ref. [15], the time dependencies of the light forces are completely antisymmetric, so that the forces produce a total modification of the atomic momentum equal zero.

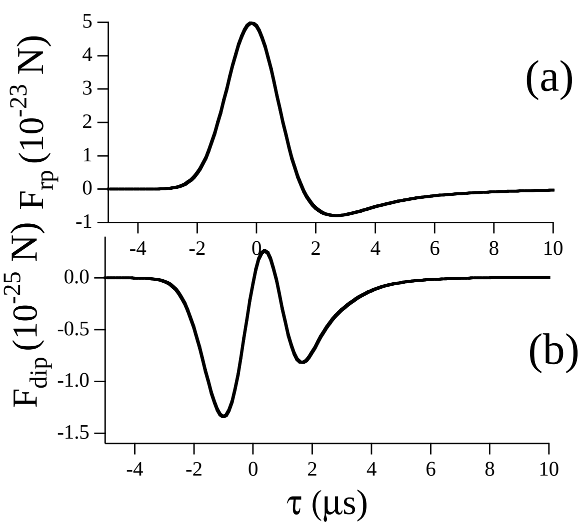

FIG. 6. Forces acting on sodium cold atoms versus the pulse-localized time τ, at $\delta_{\mathrm{c}} = \delta_{\mathrm{p}} = -A$ while the probe laser pulse of Fig. 5 enters the propagating medium (z=0). Radiation-pressure force F_{rp} in (a), and dipole force F_{dip} in (b).

V. CONCLUSIONS

We have examined the propagation of a probe pulse through a medium composed of an open three-level Λ system in the presence of a coupling laser acting on the adjacent transition of the system. Our numerical analysis confirms that also in an open system a large reduction of the probe-light velocity is obtained. The comparison with a closed system created by applying a repumping laser to the open system has demonstrated that for the parameters of the numerical analysis the velocities of the propagating pulse are equal in the compared open and closed systems. Also the amplitudes of the transmitted probe pulses are similar for the open and closed systems. This result corresponded to a set of laser parameters where the excited state occupation was very small, so that the external losses were not playing an important role on the atomic evolution. The intensity of the coupling and probe lasers were chosen in order to satisfy the requests of both EIT and excited state occupation. Furthermore a proper choice of the width of the probe pulse allowed us to satisfy the adiabaticity conditions of the atomic response, with a larger pulse transmission. We have examined the role of the laser pulse forces acting on the atoms. Those forces modify the pulse propagation and for the slowest pulses give an important contribution to the slow light velocity. The contribution of the atomic momentum to the slow light propagation can be tested using different laser propagation geometries, whence different atomic momenta for the noncoupled state of the atomic preparation.

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